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ABSTRACT

The National Council of Teachers of Mathematics (NCTM) Standards recommend that students go beyond memorizing and applying formulas to applying procedures. Students should not only understand mathematics, but also be able to do mathematics. The Core-Plus Mathematics Project (CPMP) is a comprehensive curriculum development project that develops student and teacher materials for a three year high school mathematics curriculum for all students, plus a fourth year course continuing student preparation for college mathematics. This study analyzes students' perceptions and attitudes towards the CPMP curriculum by using the CPMP Student Belief survey. Results showed that CPMP students were significantly more positive about various aspects of the curriculum and their classroom experience than students in traditional classes at the same school. Several elements of the curriculum and instructional model working together appear to explain the positive cognitive and affective results that are emerging from the CPMP field test. (ASK)

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STUDENTS' PERCEPTIONS AND ATTITUDES IN A STANDARDS-BASED HIGH SCHOOL MATHEMATICS CURRICULUM

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STUDENTS' PERCEPTIONS AND ATTITUDES IN A STANDARDS-BASED HIGH SCHOOL MATHEMATICS CURRICULUM

In the mid-1980s, the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics, the National Research Council's Mathematical Sciences Education Board, and the Mathematical Association of America began to work together in a coordinated effort to influence the direction of change in the mathematics curriculum from kindergarten through undergraduate mathematics. One document from this effort, the NCTM's Curriculum and Evaluation Standards for School Mathematics (referred to herein as the NCTM Standards), recommends that students go beyond memorizing formulas and applying formulas and applying procedures (NCTM, 1989). Students, it is recommended, should not only understand mathematics, but also do mathematics; for example, pose and solve problems, make conjectures, look for patterns, and justify and explain their mathematical thinking.

The Core-Plus Mathematics Project (CPMP or Core Plus) is a comprehensive curriculum development project funded initially by a five-year grant from the National Science Foundation. It is developing student and teacher materials for a three-year high school mathematics curriculum for all students, plus a fourth-year course continuing the preparation of students for college mathematics. The curriculum builds upon the theme of mathematics as sense-making. Throughout it acknowledges, values, and extends the informal knowledge of data, shape, change, and chance that students bring to situations and problems. Each year the curriculum features multiple strands of algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics, which are connected by fundamental themes, by common topics, and by habits of mind. The curriculum also emphasizes mathematical modeling, especially the modeling concepts of data collection, representation, interpretation, prediction, and simulation. Numerical, graphing, and programming/link capabilities of graphing calculators are assumed and capitalized on throughout the curriculum. This technology helps to facilitate the emphasis in the curriculum and instruction on multiple representations (numeric, graphic, and symbolic) and on

goals in which mathematical thinking is central. Instructional practices promote mathematical thinking through the use of rich applied problem situations that involve students, both in collaborative groups and individually, in investigating, conjecturing, verifying, applying, evaluating, and communicating mathematical ideas (Hirsch, Coxford, Fey & Schoen, 1995).

The CPMP curriculum and instructional model are described in more detail elsewhere (Hirsch, Coxford, Fey & Schoen, 1995; Schoen, Bean & Ziebarth, 1996; Hirsch & Coxford, 1997), and the textbooks, *Contemporary Mathematics in Context, Course 1 and Course 2* are now available in published form (Coxford, Fey, Hirsch, Schoen, Burrill, Hart, Watkins, Messenger, & Ritsema, 1998). Carefully developed with teacher input over a three-year period, each CPMP course is field tested in 36 high schools in Alaska, California, Colorado, Georgia, Idaho, Iowa, Kentucky, Michigan, Ohio, South Carolina, and Texas. A broad cross-section of students from urban, suburban, and rural communities with ethnic and cultural diversity is represented. Course 1 was field tested in ninth-grade classrooms in 1994-95, Course 2 was field tested in tenth-grade classrooms in 1995-96, and Course 3 was field tested in eleventh-grade classrooms in 1996-97. A great deal of quantitative and qualitative data were collected during the CPMP field test. The data includes information about various student outcome variables as measured by standardized tests and by constructed response or performance assessments, teachers' and students' attitudes and beliefs, level of implementation of the curriculum and instructional model, and specific site characteristics and experiences. Achievement outcomes are reported elsewhere (Schoen, Hirsch, & Ziebarth, 1998), and reports of several focused research studies conducted in Core Plus classrooms are cited in the appendix to this paper. This report focuses on students' attitudes and perceptions of their experiences in CPMP.

BACKGROUND

According to McLeod (1992), "...there appear to be at least three major facets of the affective experience of mathematics students that are worthy of further study. First, students hold certain beliefs about mathematics and about themselves that play an important role in development of their affective responses to mathematical situations. Second, since interruptions

and blockages are an inevitable part of the learning of mathematics, students will experience both positive and negative emotions as they learn mathematics; these emotions are likely to be more noticeable when the tasks are novel. Third, students will develop positive or negative attitudes toward mathematics (or parts of the mathematics curriculum) as they encounter the same or similar mathematical situations repeatedly." (p. 578)

Some of the main features of the Core Plus curriculum, as briefly sketched above, require many fairly abrupt changes in content, emphases and teaching methods. Changes of this magnitude and frequency require adjustments by students and teachers. While most CPMP teachers have had the benefit of professional development experiences to prepare them for the changes required of them, there is no similar advance preparation for students. They are thrust into the new classroom environment with a few initial days of orientation to this unfamiliar view of mathematics, how it is taught, and what is expected of them. Such discrepancies between students' expectations and classroom reality can elicit strong emotions, which through repeated experience may become a more stable (positive or negative) attitude or belief about mathematics and how it is taught and learned (Mandler, 1989; McLeod, 1992).

Further insight into the nature of likely discrepancies between what students encounter in Core Plus and what they expect of a mathematics class can be seen by examining beliefs students have typically brought to mathematics, especially to problem solving. A compilation of these student beliefs from the work of several researchers in the 1970s and 1980s is provided by Schoenfeld (1992).

- Mathematics problems have one and only one right answer.
- There is only one correct way to solve any mathematics problem—usually the rule the teacher has most recently demonstrated to the class.
- Ordinary students cannot expect to understand mathematics; they expect simply to memorize it and apply what they have learned mechanistically and without understanding.
- Mathematics is a solitary activity, done by individuals in isolation.

- Students who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less.
- The mathematics learned in school has little or nothing to do with the real world.
- Formal proof is irrelevant to processes of discovery or invention.

Schoenfield argues persuasively that students abstract these widely-held beliefs in large measure from their repeated classroom experiences; that is, these beliefs are consistent with the ways in which mathematics has typically been presented to students. "Furthermore, these beliefs shape their behavior in ways that have extraordinarily powerful (and often negative) consequences." (Schoenfeld, 1992; page 359)

Virtually all the beliefs in the above list are challenged in the Core Plus curriculum, suggesting rich possibilities for research on affect. That there appear to be no previous studies of students' attitudes in a Standards-based high school mathematics curriculum makes the motivation for the present study even more compelling.

METHOD

CPMP Classes

In most CPMP classrooms, students are actively engaged in investigating mathematical ideas and monitoring their emerging understanding of those ideas. For example, Course 1 teachers at midyear reported from 10% to 80% (Mean = 47%; SD = 17%) of class time was spent on small group work with the remainder being a mix of teacher-led discussion, student presentation, individual work, and student assessments. Teachers also report using a broader range of assessment techniques than is typical in traditional classes, including written and oral reports, group observations, and take-home tests to supplement the usual in-class quizzes and end-of-unit examinations.

The field test schools were encouraged to group students heterogeneously or at least in classes that included students with a wide range of achievement and interest in mathematics. Limitations at local sites did not always make this possible. Course 1 teachers' descriptions of their entering CPMP students are summarized in Table 1.

Table 1.
Percent of Teachers Giving Various Description of Their Field Test Students Upon Entering Course 1

Description	Percent
No grouping - whole range of ninth-grade students	21.5
Wide range of prior achievement but excluding best students	43.0
Wide range of prior achievement but excluding best and weakest students	12.7
More or less the typical Algebra 1 group	15.2
More or less the typical General Mathematics group	7.6

About one-fifth of the teachers reported that their classes included the full range of ninth-grade students. The most common CPMP class (as reported by 43.0% of the teachers) was comprised of students with a wide range of prior mathematics achievement and interest. Often, however, honors students were not included because they completed the grade nine course in eighth grade and moved on to a tenth-grade mathematics course in grade nine. Thus, the CPMP field test sample, as reported by the teachers, included students with a wide range of prior achievement and interest in mathematics, but accelerated honors students are probably underrepresented.

Traditional Comparison Classes

At the beginning of Course 1, eleven field test schools volunteered to pretest and posttest students in traditional, comparison classes using standardized and project-developed achievement measures. The comparison classes were comprised of 20 algebra 1, five pre-algebra, three general mathematics, and two honors geometry ninth-grade classes. Ten of these schools also administered the CPMP Student Belief Survey to their comparison students. The nature of the instruction in the comparison classes was not specified in advance, but at the end of the year comparison teachers described what transpired. For example, a variety of traditional textbooks were used. Small group work was reported to be used either not at all or less than once a week by about 80% of the comparison teachers. About 74% of the comparison teachers reported that their students used calculators more than once per week, although there is little

information about how it was used. Solving linear equations in one variable was the main instructional goal for an average of 23% of the class time for the year, with up to 50% of the time spent on this topic in some algebra I classes.

The Course 2 comparison group consisted of all students who were in the Course 1 comparison group, completed a traditional sophomore mathematics class, and completed the Course 2 posttests. Only five of the 11 schools who administered achievement tests to comparison groups in Course 1 were able to do so at the end of Course 2, and four of these five also administered the Student Belief Survey. The main reason for this drop in number is that the Course 1 comparison students enrolled in a variety of mathematics classes in their sophomore year and were difficult to locate and posttest at the end of the year. By the end of Course 3, the number of comparison students from the original pretested group that were available for posttesting was so small that a Course 3 comparison group was not feasible.

CPMP Student Belief Survey

The CPMP Student Belief Survey (SBS) is a 50-item likert scale with ten subscales, followed by an open-ended writing prompt. For Courses 1 and 2, the writing prompt was, "Think of a friend or relative in another school who is about your age. Write a letter to this person describing your experience in mathematics class this year." For Course 3, several items in the survey were changed to gather some information that was of particular interest at this point in the field test and to account for the fact that the same students had completed this survey three times in the previous two years. One change was in the writing prompt, which in the Course 3 SBS read, "Describe your experience in mathematics class this year."

The SBS was developed by the CPMP evaluation team in 1992-93 and is based partially on a mathematics belief survey used by Deborah Ball (personal communication) in some of her research in the 1980s with elementary school children. Nine of the subscales are generalized belief scales, each consisting of three to six likert items that by logical and correlational analysis appear to measure related aspects of a belief about mathematics. The scales are named to reflect what they measure as follows: Self Assessment, Challenge versus Ease, Creativity versus

Curiosity, Cooperation versus Competition, Creativity versus Rote, Genetic versus Effort, Utility, Affect, and Cooperation with Others. The tenth scale contains 15 items and is entitled, Attitude Toward Your Mathematics Course. This last scale along with the open-ended writing prompt are the focus of this paper.

The CPMP Student Belief Survey was administered as a pretest in September 1994 at the beginning of Course 1 to both CPMP and comparison students. The Attitude Toward Your Mathematics Course scale was not part of the pretest instrument, since students had not yet completed a high school mathematics class. At the end of Course 1 (May 1995) and again at the end of Course 2 (May 1996), the complete Student Belief Survey was administered to both CPMP and comparison students. Finally, at the end of Course 3 (May 1997) the slightly revised Course 3 Student Belief Survey was administered to CPMP students only.

The data analytic approach taken here is meant to be consistent with McLeod's (1992; p. 591) position, "The debate over qualitative versus quantitative research methods appears to be almost over, and the time for intelligent use of multiple research methods that fit the research problems is here." In the present study, the results are a combination of quantitative survey data from the Attitude Toward Your Mathematics Course subscale of the CPMP Student Belief Survey and qualitative data from students' responses to the writing prompts on the Student Belief Surveys each year. The approach is to present a quantitative summary of the survey data in each of several logical categories with a discussion that draws on CPMP student' written responses to help explain or elaborate findings in each category. This is followed by an attempt to synthesis of the findings across categories and in the context of previous research on affect in mathematics education.

RESULTS

For Courses 1 and 2, respectively, ten and four of the field test schools had comparison groups who completed the Student Belief Survey. Comparative results for six logical groupings of the 15 items on the Attitude Toward Your Mathematics Course scale are presented and briefly discussed below. The groupings are: Course Difficulty; Problem Solving, Reasoning and Sense

Making; Learning in Groups; Communicating Mathematics; Graphing Calculators; and Realism and General Interest. Finally, a single item that was unique to the Course 3 survey is discussed. This item concerned the role of Core Plus in keeping the students in mathematics courses for three years.

The Course 1 results are based on 834 CPMP students and 634 comparison students in eight schools, and the Course 2 results are based on 221 CPMP students and 134 comparison students in four schools. In the tables, item means are computed by assigning 1 for strongly disagree, 2 for disagree, 3 for not sure, 4 for agree, and 5 for strongly agree. The "% agree" column is the combined percent of students who either agreed or strongly agreed with that item. To test the statistical significance of the item means, a school by treatment group (CPMP versus comparison) MANOVA with the 15 items as dependent variables was run for Course 1 and then for Course 2. In each case, the multivariate school by treatment interactions as well as the school and treatment group main effects were significant ($p < .01$). Follow-up univariate school by treatment ANOVAs were then run for each item for each course, and the treatment main effect for each item and course was used to identify significant differences ($p < .05$) in treatment group means.

Course Difficulty

CPMP and comparison students did not differ significantly after either course in their average perceptions of their mathematics grades, how well they understood the mathematical ideas, and the readability of their text materials. See Table 2. As one Course 1 student wrote, "It [CPMP] challenged me a lot more than regular math, but I got a way better grade in regular math than Core Plus. But I will tell you something, you learn a whole lot more [in CPMP]." Another Course 1 student wrote, "My experience in math this year can be summed up in one sentence. It was challenging and difficult." However, some students found Course 1 to be relatively easy. "This year math was easier and more understandable & logical."

Table 2.
Items Relating to Course Difficulty with CPMP and Comparison Results by Item for Courses 1 and 2.

Items

1. *My grade is better than it was in mathematics last year.*
2. *I understood the mathematical ideas in this course.*
3. *The textbook materials in this course were difficult for me to read.*

		Course 1		Course 2	
		% Agree	Mean	% Agree	Mean
Item 1	CPMP	43.8	3.09	40.7	3.11
	Comparison	44.3	3.09	49.3	3.13
Item 2	CPMP	55.3	3.40	64.7	3.60
	Comparison	61.9	3.59	60.1	3.44
Item 3	CPMP	33.2	2.78	35.7	2.76
	Comparison	29.8	2.71	22.9	2.46

- CPMP and comparison means do not differ significantly ($p = 0.05$) on any of the items.

By the end of Course 2, some CPMP students were more articulate about their learning and their grades. For example, one wrote, "I passed with at least a C average. I figured that all you need to do is homework and be able to understand what it is you are doing. And I did it. The books and problems are easy." Another Course 2 student wrote, "It's really easy but the book is hard to understand sometimes, and everybody gets different varieties of answers, but once you get it it's basic. It's a lot more related to real life than other math classes." Finally, this student provided some further insights about CPMP. "Not all of it was easy or fun, but if you worked with it you could understand it. I think if we moved slower I would of got the class more."

Overall, these data describe CPMP as a curriculum in which the content and grading standards are at least as challenging as the traditional curriculum. Yet, many students also recognized that with work they could understand the mathematical ideas of the course. There is also some indication that CPMP students were more satisfied with their level of understanding

after two CPMP courses than after one, while the trend for the comparison students was in the opposite direction.

Problem Solving, Reasoning and Sense Making

Relative to the comparison students, CPMP students were more positive about their ability to solve problems and to reason mathematically, and they also thought CPMP helped them see that mathematical ideas make sense. See Table 3. Differences in means were statistically significant on all three items at the end of Course 2. The increased feeling of confidence seems to be closely connected for CPMP students to a recognition that the content made sense to them. As one student wrote at the end of Course 1, "I began to understand difficult ideas and methods and they didn't seem as hard." Another wrote, "I understand it now and I'm not afraid to apply myself to mathematical situations." In a similar vein, a third Course 1 student wrote, "I really think I learned more because I understood the work."

Table 3.
Items Relating to Problem Solving, Reasoning and Sense Making with CPMP and Comparison Results by Item for Courses 1 and 2.

Items

1. *This course made me feel confident that I can solve math problems.*
2. *I learned to reason mathematically.*
3. *This course helped me see that mathematical ideas make sense.*

		Course 1		Course 2	
		% Agree	Mean	% Agree	Mean
Item 1	CPMP	59.6	3.64	71.1	3.78*
	Comparison	58.3	3.59	55.6	3.41
Item 2	CPMP	59.5	3.63	68.8	3.77*
	Comparison	51.7	3.45	53.0	3.44
Item 3	CPMP	60.1	3.60	64.7	3.71*
	Comparison	57.0	3.53	51.1	3.30

* This mean is significantly greater ($p < 0.05$) than the mean of the other group on this item.

This theme of understanding leading to improved confidence and ability to solve problems and reason mathematically can be seen at the end of Course 2 as well. One Course 2

student wrote, "...it [CPMP] made it much easier to learn and figure out the problems and how to apply [the mathematics]." Another echoed this sentiment, "...the most important thing was that it [CPMP] made me think and understand what I was doing." One Course 3 student was probably the most articulate about understanding and its related pay-offs, "I learned to comprehend at a technical level, with that I was able to view mathematical meanings and reasoning with an open mind, therefore allowing me to further my growth in a field I struggled to get by in."

Learning in Groups

CPMP developers recommend that the investigations be completed by students working in small groups or pairs. Teacher survey and classroom observation data reported earlier in this paper indicates that, on average, just under half (but in some classes as much as 80%) of class time in Course 1 field test classes was spent in small group or pair arrangements, so small cooperative group instruction is an important component of the CPMP instructional model.

Students were directed to skip the items given in Table 4 if they did not use group work in their classes, and about 19% of the comparison students did so. Both CPMP and comparison students, on average, enjoyed group work and believed it helped them learn mathematics. The only statistically significant difference between CPMP and comparison means occurred at the end of Course 1 when the CPMP students agreed more strongly than comparison students that group work helped them learn mathematics.

Table 4.
Items Relating to Learning in Groups with CPMP and Comparison Results by Item for Courses 1 and 2.

Items

1. *Group work helped me learn mathematics.*
2. *The group work was enjoyable.*

		Course 1		Course 2	
		% Agree	Mean	% Agree	Mean
Item 1	CPMP	63.5	3.67*	79.2	4.07
	Comparison	59.3	3.61	76.5	3.95
Item 2	CPMP	69.6	3.84	76.9	3.97
	Comparison	70.1	3.84	81.9	4.11

* This mean is significantly greater ($p < 0.05$) than the mean of the other group on this item.

The enjoyment of working in groups and its value for social learning is expressed well by this Course 2 student, "I don't think I have ever had so much fun doing problems and solving them in a group. I learned a lot of how to work with people." The experience of seeing how other people attack problems and the mutual supporting of efforts by group members seems to be the key to learning in groups. As this Course 1 student wrote, "Math was pretty cool this year because we got to be in groups...the groups were helpful because if I didn't get a problem there was a chance someone else knew what it was." Another student (in Course 3) enjoyed the role of helper for others in the group, "I also had fun working with groups and helping them get the answers." On the other hand, a few CPMP students (about 15%) did not like working in groups, mainly because some students did not contribute enough or they perceived the learning to be inefficient. As this Course 3 student wrote, "I would have learned better in this course if I would have learned it from the teacher the first time instead of learning it wrong in groups."

Communicating Mathematics

In CPMP, students in groups read through the mathematical material and investigation questions. They also discuss the mathematics orally in their groups and often present what their group learned to the whole class. The CPMP assessments require more written description of the student's mathematical thinking than most traditional assessments. In addition, many CPMP

teachers make occasional assignments that involve oral or written reports. Given all this opportunity to communicate mathematics, perhaps it is not surprising that CPMP students at the end of each course agreed more strongly than comparison students that their mathematics course made them better at both talking and writing about mathematical ideas. See Table 5.

Table 5.
Items Relating to Communicating Mathematics with CPMP and Comparison Results by Item for Courses 1 and 2.

Items

1. *This course made me better at talking about mathematical ideas.*
2. *This course made me better at writing about mathematical ideas.*

		Course 1		Course 2	
		% Agree	**Mean**	**% Agree**	**Mean**
Item 1	CPMP	53.2	3.41*	68.2	3.72*
	Comparison	47.1	3.31	42.9	3.09
Item 2	CPMP	55.9	3.47*	66.5	3.71*
	Comparison	46.7	3.30	40.6	3.06

* This mean is significantly greater ($p < 0.05$) than the mean of the other group on this item.

In spite of the consistently positive perceptions of communicating mathematics on the attitude survey, students did not often mention this aspect of CPMP in open-ended written descriptions of their experience in mathematics class suggesting that most did not see it as a particularly important aspect of CPMP. One Course 1 student saw writing as generally important. "The writing helps a lot considering there's a lot of it in life..." Another said, "I have learned to express my mathematical ideas in words which before this class I was not able to do." A third Course 1 student did not like writing, "The class is fine. But I can't stand writing in complete sentences. All the other math classes just have to write the answer. We have to, and in complete sentences."

Graphing Calculators

The pattern of student attitudes about the graphing calculator is similar to the pattern for group work. Both CPMP and comparison students consistently indicated that they enjoyed using graphing calculators and, a little less consistently, that they learned more mathematics by using

the calculator. Students were instructed to skip these items if they had not used calculators in their mathematics class, and about 11% of comparison students did so. On the other hand, all CPMP students used graphing calculators which were intended to be a natural and always available tool. Results on the two graphing calculator items are given in Table 6. The only statistically significant difference was at the end of Course 1 when CPMP students agreed more strongly than comparison students that they learned more mathematics by using the graphing calculator.

Table 6.
Items Relating to Graphing Calculators with CPMP and Comparison Results by Item for Courses 1 and 2.

Items

1. *I learned more mathematics by using the calculator.*
2. *I enjoyed using the calculator in this course.*

		Course 1		Course 2	
		% Agree	Mean	% Agree	Mean
Item 1	CPMP	57.0	3.55*	71.5	3.95
	Comparison	50.6	3.45	69.6	3.85
Item 2	CPMP	69.9	3.82	85.1	4.28
	Comparison	72.8	4.01	86.5	4.33

* This mean is significantly greater ($p < 0.05$) than the mean of the other group on this item.

Most of the students who wrote about the calculator mentioned that they had learned more. Some specific content references were made by several students. "The calculators taught me a lot about graphs and tables" and "I learned by using a calculator to graph and find the y-intercepts." Other comments like the following were more general. "...we use an advanced calculator which helps us expand our knowledge of math," "The calculator was helpful, especially when you're making graphs or dealing with a lot of data and statistics," and "I learned so much on the calculator." The few students who expressed negative opinions about the calculator appeared to be concerned that they would become too dependent on it. As one student said, "In real life, I might not have a calculator around wherever I go."

Realism and General Interest

Courses 1 and 2 The three items in Table 7 were grouped primarily because realism of the problem contexts in CPMP is the variable that students most frequently connect to their interest in the course. Realistic problems clearly make a mathematics course more interesting for many students, as a Course 1 student wrote, "...we do more realistic math problems and that makes it most interesting." Finally, to connect the third item to the first two, students are likely to want to take another math course taught in the same way if they have found that this one was interesting. As a Course 3 student wrote, "I do plan to take this type of course next year, because it's more realistic than the regular math class."

Table 7.
Items Relating to Realism and General Interest with CPMP and Comparison Results by Item for Courses 1 and 2.

Items

1. *Most of the problems in the course are realistic.*
2. *This course made the mathematical ideas interesting to me.*
3. *I want to take a math course taught in the same way next year.*

		Course 1		Course 2	
		% Agree	Mean	% Agree	Mean
Item 1	CPMP	60.4	3.59*	76.5	3.95*
	Comparison	45.9	3.27	47.8	3.17
Item 2	CPMP	50.8	3.33	70.1	3.74*
	Comparison	50.0	3.35	41.4	3.07
Item 3	CPMP	53.3	3.42	75.0	4.05*
	Comparison	51.4	3.49	43.0	3.13

* This mean is significantly greater ($p < 0.05$) than the mean of the other group on this item.

First, to elaborate on the realism of the problems, a Course 1 student wrote, "I've really enjoyed learning problems that I might actually need to use in real life." Another wrote, "The problems are realistic and make sense. I never found myself asking questions, like 'When will I ever use this in life?'" A third Course 1 student related the realism to ease of understanding, "It had a lot of real life situations which made the math easier to understand." A Course 2 student

related the integration of topics and realism, "...an integrated kind of math that takes math ideas and teaches them to us in a realistic way." A second Course 2 student saw the realism as good preparation for the future, "...the problems are realistic...definitely helps prepare students for their future." A Course 3 student described the value of the course in some detail, "This was my first year in [CPMP]...before this I took algebra, geometry and advanced algebra...for the first time I could relate math to everyday life. Learning geometry and things like tan, sin, cos, actually made more sense than learning it in traditional math."

Second, to elaborate on the general interest level of the course, a Course 1 student wrote, "This math class is the first class that I have took [sic] that I like in a long time." Other students expressed a general pleasure with Courses 1, 2, and 3, such as, "...how much fun I had this year in math class," "It was a funner way of doing math, I actually looked forward to coming to class," and "It has been a delightful experience throughout." Besides the realism of the problems, some students mentioned other interesting aspects of the course. Some examples are, "there's more hands on activities and that makes it fun to learn," "this course makes you think for yourself and solve problems for yourself," and "...this course makes us think. It is very interesting."

Third, to elaborate on the decision to take a math course taught in the same way next year, several students simply said they would do so, "I'm going to take it next year," "I really hope that they teach math this way next year," and "I will probably take this course next year. I really like the teaching method." A quote given earlier attributed the decision to take next year's CPMP course to the realism of this year's course. Another student, this one in Course 3, attributed the decision to having learned a lot, "I learned a lot and I feel knowledgeable. I enjoy math and plan to take a fourth year of it."

A few CPMP students answered negatively concerning their interest in the Core Plus course and in continuing in the next course. Written comments suggest that negative responses on this item may have been motivated by one of three issues: (1) a worry that there was not enough work on algebraic skills (e.g., "I didn't take algebra so now I'm stuck in this stupid

course."; (2) finding the Core Plus course to be too difficult (e.g., "I only understood maybe 2 of the books and we had at least 10 different books. My grade improved from last year, but I think this class isn't a good class for me. It's mostly hard stuff and it gets frustrating and confusing."); or (3) finding the course uninteresting or boring (e.g., "I wish it wasn't so rythmatic [sic]. We do the same thing every day, boring....but I did learn more than last year.").

Course 3 Related to the issue of wanting to continue in more Core Plus courses is the following item, administered only on the Course 3 Student Belief Survey: *It is mainly because of Core Plus that I took a third year of high school math.* Of the 1,944 Course 3 students completing this survey, 297 (15.3%) strongly agreed and 223 (11.5%) agreed with this statement. Thus, over one in four of the Course 3 students attributed their being in a third year of mathematics mainly to the Core Plus curriculum.

DISCUSSION

Students perceive the Core Plus curriculum as quite difficult, at least as challenging as traditional college-prep mathematics courses. Nevertheless, Core Plus students were significantly more positive about various aspects of the curriculum and of their classroom experience than students in traditional classes in the same schools. A common perception of students is that Core Plus is challenging and makes them think, but with effort it is possible for them to understand the ideas and their applications. Indeed, achievement results available to date for the wide range of field test students in Core Plus classes are strong, especially in areas of understanding, reasoning and problem solving (Schoen, Hirsch, & Ziebarth, 1998).

It seems reasonable to conclude that affective and cognitive factors are working together to lead to the positive findings in each domain. The following hypotheses draw on the CPMP field test data presented here and in Schoen, Hirsch, & Ziebarth (1998), as well as on classroom observations and interviews with teachers and students. Each hypothesis is meant to suggest an area in which more research is needed.

- *Students of wide mathematical backgrounds are able to access and make sense of the challenging Core Plus curriculum mainly because of the contextualized entry points into new ideas.*

  Such an entry point makes use mainly of common language to explain content and draws upon contextual situations that are often familiar to students through their experiences outside school. Symbols and symbol operations are encountered at first as representing variables in these familiar contexts, and gradually the symbol work becomes more formal and de-contextualized.

- *Significant improvement in conceptual understanding, problem solving and reasoning are directly connected to the positive attitudes of the students in these areas.*

  This hypothesis is simply asserting that the positive cognitive and affective outcomes in Core Plus arise from related sources that tend to support and enhance one another. More specifically, positive cognitive outcomes give rise to positive affect in that improvements in conceptual understanding, problem solving and reasoning as documented in the field test are likely to make students more positively disposed toward these areas of the curriculum. On the other hand, positive affective outcomes like greater interest and confidence lead to more effort, hence, to more learning. In support of the previous statement, results presented in this paper show that many Core Plus students were stimulated by the challenge of mathematical sense making, reasoning and problem solving and felt confident that they could be successful.

- *For many students, learning in groups contributes to the positive cognitive and affective outcomes in conceptual understanding, problem solving, and reasoning.*

  On the cognitive side, for many students, fellow group members provide a source of aid and interaction that is less threatening than asking the teacher. Seeing how other students attack problems and try to make sense of new concepts provides students with opportunities for insights into their own and their classmates' cognition that are not available in a teacher-centered classroom. On the affective side, shared false starts, errors and eventual success

while struggling through complex, but accessible, tasks reinforces students' positive attitudes and confidence. There is obvious affective reinforcement associated with eventual success on a challenging task.

- *Writing about and reporting orally on mathematical topics provides students with the opportunity to reflect on their own mathematical thinking. Being required to write and report orally emphasizes the belief that mathematics is a subject in which thinking is important and potentially fruitful.*

  Reflecting on one's own thinking is well known to be an aid to cognition (Schoenfeld, 1992), and the belief described in the second sentence above is one that is strongly valued by mathematics educators at all levels.

- *Using graphing calculators appropriately is an aid to learning and applying many mathematical topics and also has a positive impact on many students' confidence and tenacity as they do mathematics.*

  Graphing calculators are especially useful in developing students' ability to understand functions in their various representations (symbols, graphs, tables, and contextualized settings). The statistical capabilities of many of these calculators are also very helpful. The data presented in this paper and other class observation and teacher perception data from the Core Plus field test support the hypothesis that students are more tenacious and confident of success when they have the calculator available.

- *The perceived realism of the contexts for investigations and problems is perhaps the strongest contributor to students' high levels of interest in continuing to enroll in Core Plus courses.*

  At first glance, this hypotheses may seem to conflict with the notoriously negative attitudes toward, and lack of success with, word problems of students in traditional curricula (Sowder, 1989). Introducing more, and more challenging, word problems into such an environment would hardly seem to have potential to improve students' attitudes or achievement. The key to resolving this apparent conflict, we believe, is that traditional word problems are usually

addressed after students have supposedly mastered the symbol manipulations that will solve the word problem. In fact, research clearly shows that the traditional sequence of symbol manipulation skills, especially in algebra, leaves many students with little understanding of the meanings of the symbols and their operations and hence of how they connect to real contexts such as those presented in word problems. Traditional approaches to teaching the connection between symbols and word problems often use cue words, dictionary-like translation of phrases to symbols, or simply mimicking many solutions to similar problems. Unfortunately, these approaches are not powerful or generalizable mainly because they try to circumvent the need for students to understand the symbol system, the problem context and the modeling connection between symbols and context. Thus, many students in the traditional curriculum have no meaningful way to attack word problems, hence, their failure and negative attitudes in this domain.

On the other hand, Core Plus uses the fact that well-chosen application contexts are what is initially understandable to students, and so these contexts serve as the vehicle for learning the meanings of the symbols and operations. The meanings of the symbols and operations are then abstracted from experiences with many contexts of related mathematical structure. Students do not find their way to learning blocked prematurely by symbols that are completely mysterious to them, but rather they focus on understandable contexts and make use of symbols to model the contexts in ways that make sense. Symbol manipulation for its own sake is practiced but only after experience with the symbols and procedures in a variety of contexts with similar mathematical structures. Gradually, as students' understanding of and facility with symbols matures, the symbolic work becomes more formal and de-contextualized.

FINAL NOTES

The findings of this study suggest that Core Plus students see frequent counterexamples of the commonly-held beliefs compiled by Schoenfeld (1992) and described earlier in this paper. Particular instances are the following.

- Having often seen in group work the best students struggle with the mathematical ideas, it seems unlikely that students will come to believe that those who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less.
- Many of the Core Plus investigations and problems have more than one solution path and more than one right answer, and the group investigations usually elicit at least the intended variety of both. The discussions which accompany these different solutions are often particularly rich learning opportunities.
- A major goal of the Core Plus curriculum is for students to view mathematics as useful for making sense of quantitative situations, not just as symbol manipulation procedures to be learned mechanistically.
- Far from seeing school mathematics as having little or nothing to do with the real world, about 75% of the Core Plus students perceived the investigation and problem contexts as realistic.

Several elements of the curriculum and instructional model working together appear to explain the positive cognitive and affective results that are emerging from the Core Plus field test. More research is needed to better understand these elements and their interactions in classrooms that use Core Plus or other Standards-based curricula.

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